Reply to "Comment on 'Maximal planar networks with large clustering coefficient and power-law degree distribution' "

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We give a brief review on the analytic approaches for finding the degree distribution. The method used in the comment (master-equation) and the one in the original paper (rate-equation) [T. Zhou, G. Yan, and B. H. Wang, Phys. Rev. E **71**, 046141 (2005)] are two mainstream methods. The former is more accurate, and the latter is more widely used since it can solve some complicated problems that cannot be easily solved by the former approach. The analytic forms of finding the degree distribution obtained by the above two methods have the same asymptotic behaviors.

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In a recent paper, we proposed a network model, named *random Apollonian networks* (RANs for short), which can simultaneously display small-world effect and scale-free property [1]. By using the rate-equation approach, we obtained the solution of degree distribution

$$p(k) \sim k^{-3}, \tag{1}$$

where k denotes the degree and $p(k)$ is the probability function. The analytic result agrees with the simulation very well (see Fig. 4 in Ref. [1] for details). Wu *et al.* oppugn the validity of this theoretical approach and by using the master-equation approach, they obtained a more accurate result [2]

$$p(k) = \frac{2m(m+1)}{k(k+1)(k+2)}, \tag{2}$$

where $m=3$ is the degree of a node at the time it enters the system.

There are various analytic approaches aiming at the dynamical properties of the scale-free models [3]. First, Barabási *et al.* proposed the so-called continuum theory [4]. And then, almost at the same time, Dorogovtsev *et al.* [5] and Krapivsky *et al.* [6] introduced the *master-equation* and *rate-equation* approaches, respectively. The former is used in the present comment [2] (Eq. (4) in Ref. [2] is completely the same as Eq. (90) in Ref. [3]), and the latter is used in Ref. [1]. Although there are slight differences between the master-equation and rate-equation approaches, these two approaches offer the same asymptotic results and, thus, can be used interchangeably. For example, Eqs. (1) and (2) display the same asymptotic behavior for large k. Sometimes, the master-equation approach can get a more accurate result than that of rate-equation approach, but the rate-equation approach is simpler and more easily solved; thus, it is more suitable for some more challenging tasks, for example, obtaining the clustering coefficient [7] and assortativity [8]. Therefore, it is not proper to say the theoretical approach (rate-equation approach) used in Ref. [1] is wrong, while the one (master-equation approach) in the Comment [2] is right. In addition, Wu *et al.* argue that the probability a node with degree k will link to the new node is not $\frac{k}{N_\Delta}$. However, since there are k triangles containing a k-degree node and the total number of triangles is N_Δ, the corresponding probability a randomly selected triangle containing a given k-degree node is clearly and undoubtedly $\frac{k}{N_\Delta}$, which is also an essential property of the model.

Furthermore, the theoretical approach in Ref. [1] is also used in the generalized cases of RANs (named simplex triangulation networks [9] or high-dimensional RANs [10]). The analytic solution of power-law exponent $2+\frac{1}{d-1}$ for the d-dimensional case is obtained, which agrees very well with the simulation.

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